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Modeling and Flight Control of Large-Scale Morphing Aircraft

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As morphing is an emerging topic of interest in aircraft research, the following article provides a review of the subject, with specific focus on modeling and flight control of large-scale planform altering flight vehicles. Our discussion proceeds in a fundamental manner to demonstrate that, although design methods for rigid aircraft have become highly developed, the consideration of morphing necessitates further investigation into the typically disparate fields of dynamic modeling, aerodynamic theory, and flight control theory. To clarify these points, the equations of atmospheric flight are derived in a general form, methods of integrating the aerodynamic forces are examined, and we distinguish between various approaches and methods of flight control.

Nomenclature

$\boldsymbol{A}$, $\boldsymbol{B}$	=	matrices in the linear state equations
B	=	generalized constraint matrix
$B^{\perp}$	=	orthogonal complement of B
C, D	=	matrices of the kinematic equations
C_M	=	pitching moment
C_{M_α}, C_{M_q}, $C_{M_{\delta_e}}$	=	pitching moment due to α, Q, δ_e
$\mathcal{D}$	=	spacial domain of aircraft body
$\boldsymbol{F}$	=	externally applied force
F_B	=	body-fixed reference frame
F_I	=	inertial reference frame
F_K	=	center of mass reference frame
f	=	generalized force
g	=	gravitational constant
J	=	second inertial moment
M	=	generalized mass array
m	=	total aircraft mass
$\bar{m}$	=	total morphing body mass
N	=	generalized aerodynamic function
Q	=	pitch rate
q	=	generalized coordinate
$\boldsymbol{R}$	=	location of F_B relative to F_I
$\boldsymbol{r}_p$	=	location of p relative to F_B
$\underset{\sim}{S}$	=	first inertial moment about F_B
$\bar{S}$	=	first inertial moment of $\int \boldsymbol{r}_p$ about F_B
$\boldsymbol{T}$	=	thrust vector
t	=	time
u	=	planform coordinate
$\boldsymbol{u}$	=	control state vector
u_R	=	planform reference input
$\boldsymbol{V}$	=	body-fixed translational velocity
v	=	planform control dynamics
$\boldsymbol{v}_p$	=	time derivative of $\boldsymbol{r}_p$
w	=	planform generalized speed
$\boldsymbol{w}$	=	reduced control vector
$\boldsymbol{x}$	=	state vector
$\boldsymbol{x}_i$	=	state vector at time t_i
y	=	generalized velocity
$\boldsymbol{z}$	=	reduced state vector
α	=	angle of attack
Δ	=	vector increment
δ	=	function variation
$\boldsymbol{\delta}$	=	vector of control surface deflections
η	=	atmospheric conditions
Θ	=	rotational elements
θ	=	pitch angle
λ	=	array of Lagrange multipliers
ν	=	number of mass elements of morphing body
π, γ	=	coefficient matrices of local kinematics
ρ	=	mass density
τ	=	generalized constraint force
τ_c	=	internal controlled constraint
τ_f	=	constraint due to friction
φ	=	generalized applied force
φ_a	=	generalized applied aerodynamic force
φ_T	=	generalized applied aerodynamic thrust
φ^*	=	generalized inertial force
$\boldsymbol{\omega}$	=	body-fixed rotational velocity
0	=	column vector of zeros
1	=	identity matrix

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I. Introduction

AIRCRAFT design has evolved at an extraordinary rate since the first manned flight in 1903. In only a century engineers have constructed aircraft that can travel at many times the speed of sound, traverse the Earth's circumference without refueling, and even breach the atmosphere into space. Modern aircraft are capable of large payload transport, extreme maneuverability, high speeds, high altitudes, and long ranges; they are capable of stealth, vertical takeoff and landing, and unmanned flight. Of course, no one design possesses all of these qualities. In fact, aircraft designs may be radically different depending on the operational requirements. To service the ever-growing mission requirements of the military, since 1970 no less than 36 new aircraft have been designed, and all but three have been flight tested, under contract by the U.S. Department of Defense. In the year 2000 there were 12 active aircraft production lines servicing the U.S. arsenal, with one producing unmanned aircraft [1].

The reason for varying aircraft designs is that, due to atmospheric interactions, mission capability is dictated predominantly by structural geometry. As a result, although a rigid aircraft may be designed to perform exceptionally in some given flight regime, another can be found in which the performance is relatively poor. An apparent remedy to this situation is to provide the necessary shape altering capability that would enable a single aircraft to encompass a larger range of performance.

Morphing aircraft are flight vehicles that alter their shape to effectuate a change in either mission performance and/or to provide control authority for maneuvering. Aircraft with morphing capability promise the distinct advantage of being able to fly multiple types of missions and to perform extreme maneuvers not possible with conventional aircraft designs. However, as almost all current aircraft designs are centered around a fixed planform arrangement, the consideration of morphing calls for an investigation into several research areas including aerodynamic modeling, nonrigid body dynamics, and flight control theory. Of particular importance is a means of integrating these topics to provide a more unified approach to control design.

An aircraft capable of significant planform reconfiguration requires a flight control design capable of high performance while maintaining stability in the presence of (or with the aid of) large variations in mass distribution and applied aerodynamic forces. Although there are numerous applications of morphing technology in the literature, there is a notable lack of published work on fundamental approaches to modeling and controlling such aircraft designs. This is, however, not unexpected since the difficulties that are inherent in the current subject are fundamental problems in their own right: namely, useful approximations of the forces applied to complex structures moving through the atmosphere and control of uncertain, nonlinear systems.

In this work we discuss important topics of morphing aircraft research, with a specific focus on modeling and flight control design of large-scale planform altering flight vehicles. No new theory is introduced; but rather we collect a large portion of the topics relevant to morphing applications and consider the extension of the associated concepts and methods to the current subject. Furthermore, we direct the reader to areas where new developments would contribute substantially to the field. The following section provides an overview of previous research to provide perspective on the topics later presented. Following this discussion, a general method for formulating the dynamic equations is given along with the underlying reasons for the given approach. Methods of formulating state-dependent aerodynamic forces and moments are discussed in the next section. The final section covers flight control methods and identifies the differing approaches and methods applicable to morphing aircraft control.

II. Morphing Aircraft Concepts

The first significant planform altering aircraft design was the variable sweep wing. Both theory and experiments had shown that, although the straight wing was sufficient for most tasks, a swept wing is more ideal for high-speed flight, particularly at supersonic speeds [2]. The first aircraft flown with the variable sweep design was the X-5 in 1951; later on, the F-111 and F-14 were also equipped with variable swept wings. Ultimately, however, the additional weight of the mechanisms to accommodate the wing changes were found to be costly with regards to fuel efficiency. It is this problem of weight that continues to be the largest obstacle in designing a feasible shape changing aircraft. Nonetheless, designers have continued to incorporate shape changing technology, albeit much smaller, into aircraft wings. Commercial transport aircraft, as well as military fighters such as the F-16 and F-18, alter chord and camber shape using leading and trailing edge flaps.

Wing shape changes currently in practice, although beneficial, provide only marginal improvements within a given flight regime. They cannot, for instance, transform a fighter into a vehicle capable of long range and endurance. As an example, the F-16 has an aspect ratio of 3.2 while the RQ-4A Global Hawk has an aspect ratio of 25. Current designers seek to create the ability to alter the critical geometrical properties that would enable a single vehicle to accommodate a much larger range of mission requirements. Such geometry changes are much more substantial than conventional designs. To enable such large structural changes, while overcoming weight restrictions, requires significant improvements in both materials and actuator technology relative to what is currently applied in aircraft construction.

A. Early Concepts

Some of the earliest concepts in morphing technology were a result of separate developments. The first was in the field of aeroelasticity where, as noted by Barrett [3], researchers began to investigate the benefits of actively tailoring the response of aeroelastic structures for improved control authority [4,5]. The second development was a significant increase in the application of and improvements in active material technology. Given their impressive energy density, materials such as piezoelectric/electrostrictive ceramics and shape memory alloys (SMAs) were viewed as suitable replacements for conventional actuation devices. Subsequently, active material technology applied to flight vehicles has included the development of a bimorphpiezoelectric hinged flap [6], a trailing edge flap actuator for helicopter rotors [7,8], and flexspar actuators for missile fins [3,9].

As a result of previous successes, government sponsored research programs have sought to investigate shape changing aircraft technology. The first of these was the Defense Advanced Research Projects Agency/NASA/Air Force Research Laboratory (AFRL)/Northrop Grumman sponsored Smart Wing Program [10–14]. Begun in 1996, the Smart Wing program consisted of two phases culminating in wind tunnel testing of an aircraft wing outfitted with a SMA actuated, hingeless, smoothly contoured trailing edge. Some benefits were a 15% increase in rolling moment and 11% increase in lift relative to the untwisted conventional wing.

The second program, started in 1996, was the Active Aeroelastic Wing (AAW) program, a joint program sponsored by DARPA/NASA/AFRL/Boeing Phantom Works [15–17]. The goal of this program was to demonstrate the advantages of active aeroelastic wing technology. The final result of the project was the flight testing of a full-size aircraft (F/A-18) equipped with flexible wings. Roll control was achieved by a differential deflection of the inboard and outboard leading-edge flaps. In addition to significant aerodynamic improvements, it was shown that active aeroelastic wing technology could reduce aircraft wing weight of up to 20%.

Yet another program is the Active Aeroelastic Aircraft Structures (3AS) project in Europe [18]. Under this program various active aeroelastic concepts have been developed and demonstrated. Of note, Amprikidis et al. investigated internal actuation methods to continuously adjust wing shape while maintaining an optimal lift-to-drag ratio [19–21].

B. Current Trends

The most current U.S. sponsored initiative is the DARPA Morphing Aircraft Structures (MAS) program. This program was initiated with the goal of researching larger wing shape changes than have been previously investigated. Under the MAS program a *morphing* aircraft was defined as a multirole platform that

1) changes its state substantially to adapt to changing mission environments;

2) provides superior system capability not possible without reconfiguration;

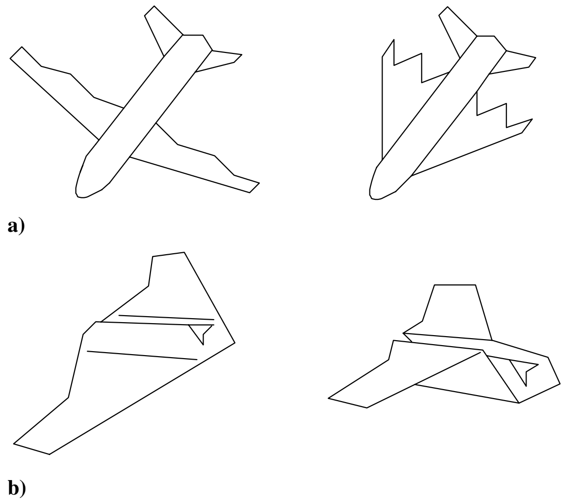

Fig. 1 Current trends in morphing aircraft design: a) NextGen Aeronautics, b) Lockheed Martin.

3) uses a design that integrates innovative combinations of advanced materials, actuators, flow controllers, and mechanisms to achieve the state change.

The program originally funded three main contractors: NextGen Aeronautics, Raytheon Missile Systems, and Lockheed Martin. Several university partners were also included in the program resulting in numerous theoretical and experimental studies of material applications and methods of altering structural geometry to obtain improvements in flight performance. One of the first concepts from this program was an extension of leading and trailing edge control surfaces to fully adaptive wings, that is, a wing capable of changing its camber shape at each segment along the span of the wing. Like a bird's wing, such a design would be devoid of control surfaces, achieving flight control authority by changing wing shape. Of note, Gern et al. performed a structural and aeroelastic analysis of an unmanned combat aerial vehicle (UCAV) with morphing airfoils [22,23]. Petit et al. performed similar evaluations [24] and Bae et al. [25,26] investigated the 2-dimensional aeroelastic effects of a variable camber wing. Because shaping a wing to achieve a desired control authority has nonunique solutions, several researchers have investigated optimizing wing shape to minimize actuator energy [27–29]. Others have investigated the methods to actuate morphing wing sections; novel methods such as *compliant mechanisms* have been studied extensively [30–32].

While research continues in all areas of morphing applications, the MAS program has recently focused on the development of aircraft that are capable of more significant planform alterations. Stated goals include a 200% change in aspect ratio, 50% change in wing area, 50% change in wing twist, and 20-deg change in wing sweep. Furthermore, it is expected that the weight of the resulting wing should be no greater than that of a conventional aircraft. Such a design could be optimized for a given flight regime, such as loiter or takeoff. The magnitude of the geometry changes coupled with the weight restrictions distinguishes a morphing aircraft from previous designs such as the sweep wing.

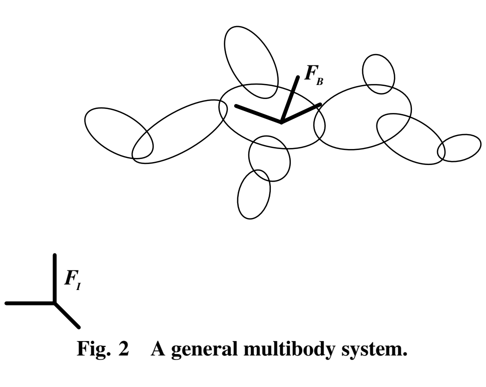

Fig. 2 A general multibody system.

Efforts to create these types of morphing aircraft are currently ongoing. The concept morphing aircraft designs for NextGen and Lockheed [33] are depicted in Fig. 1. Supporting research at the university level has included investigation into various wing morphing designs. Bae et al. performed an aeroelastic analysis of a variable-span wing for cruise missiles [34,35]. Blondeau et al. developed an inflatable telescopic spar and performed a wind tunnel test of a variable-span wing [36], and Neal et al. developed a scaled morphing aircraft for wind tunnel testing capable of variable span, sweep, and wing twist [37].

III. Morphing Dynamics

We consider a general multibody system, depicted in Fig. 2, where none of the members are necessarily rigid. The configuration of the system can be quantified by n generalized coordinates, accumulated in the one-dimensional array q. The kinematic differential equations may be expressed in the general form

$$\dot{q} = C(q, t)y + D(q, t) \tag{1}$$

where y is a $p \times 1$ array of generalized speeds (or quasicoordinates). The dynamics of the system can be expressed in the form

$$M(\dot{y}, y, q, t) = f(q, y, t) \tag{2}$$

where M is an $n \times 1$ generalized mass array, and f is an $n \times 1$ array of generalized forces. We divide f into its components

$$f = \varphi^* + \varphi + \tau \tag{3}$$

where φ^* are those components that result from representing the dynamics in a noninertial reference frame, (i.e., inertia and centrifugal terms), φ are externally applied forces, and τ accounts for constraints due internal interaction forces.

A reference frame F_B is attached to a rigid part of the main body (likely some portion of the aircraft's fuselage) and is confined to both rotate and translate with the body. The "gross motion" of the system, the motion by which flight performance and stability is evaluated, is considered to be that of F_B with respect to the inertial reference frame F_I. This motion is quantified by the Cartesian vector $\boldsymbol{R}$, which locates the origin of F_B, and Θ, which specifies the orientation of the basis vectors. The number of orientation coordinates will be either three or four depending on the parameterization. In flight mechanics, it is standard to use the 3-2-1 set of Euler angles, although this is not always the case [38].

The dynamic equations are formulated with respect to the single body-fixed reference frame F_B, as opposed to multiple frames attached to each body. The reason for this choice is that we are considering bodies that are composed of many interconnected parts, each undergoing large complex relative motions. As such it would be difficult to assign the placement of the frame's origin. For a system consisting solely of rigid bodies the choice of coordinate frames and the derivation of the equations of motion is more straightforward [39,40]. There are several methods in nonrigid body dynamics to locate "floating reference frames" for complex motions, including those that permit decoupling of the rigid body and internal motions [41]. However, as discussed by Meirovitch, for aircraft applications such reference frames introduce unnecessary complexity due to the nature of the applied atmospheric forces [42]. Furthermore, most aerodynamic routines determine the loads for the entire body as a force and moment couple at a single reference point, typically the center of mass. In this respect, another reference frame of consideration is one that translates with the mass center of the entire body (but at each instant oriented with F_B). Sometimes referred to as Koenig's frame, denoted F_K, this frame decouples the translational rigid-body motions from the internal motions [43].

The most general formulation of the dynamic equations would consider the rigid-body motions of the main body, elastic deformations, and controlled motions of the system (i.e., morphing). Elastic effects can be significant due to the coupling between the aerodynamic force distribution and the structural dynamics. A comprehensive review of the aeroelastic response of flight vehicles was performed by Mukhopadhyay [44]. Perhaps the most enlightening work in regards to aeroelastic flight analysis was that of Milne [45] in which the longitudinal equations were examined in the context of stability. Aeroelastic effects are typically approached by either coupling the fluid mechanics equations with the structural equations or by integrating the structural problem with methods of aerodynamic theory to account for the force distribution. The former approach leads to highly complex expressions generally only suitable for numerical studies. The latter approach was recently used by Meirovitch and Tuzcu [46–48], in which the flexible body equations were used in conjunction with aerodynamic strip theory.

Here we are presently concerned with large controlled deformations and do not wish to complicate matters by considering aeroelastic effects, where it would be necessary to append an additional set of equations of the form provided in [49]. Such effects, however, are crucial to such applications as variable camber morphing, as recently demonstrated by Bae et al. [26].

A. Dynamic Formulation

The mapping of the basis vectors of F_B into those of F_I is provided by the orthogonal operator $C_1(\Theta)$, where $C_1^{-1} = C_1^T$. Let $\boldsymbol{V} := C_1\dot{\boldsymbol{R}}$ and $\boldsymbol{\omega} := C_2\dot{\Theta}$, where $C_2(\Theta)$ will be determined by the rotation parameterization (note that $C_2^{-1} \neq C_2^T$). If, instead, F_K is used to define motion then the transformation operators are identical.

In addition to the main body motion, let the remaining nonrigid motions of the aircraft be described by the $m \times 1$ coordinate array u such that the position with respect to F_B of an arbitrary element, given by the Cartesian vector $\boldsymbol{r_p}$, can be expressed in the form $\boldsymbol{r}_p = \boldsymbol{r}_p(u, t)$, $p = 1, \ldots, \nu$. The time derivative, $\dot{\boldsymbol{r}}_p := \boldsymbol{v}p$, can be written as

$$\boldsymbol{v}_p = \pi_p(u, t)w + \gamma_p(u, t) \tag{4}$$

which satisfy the kinematic equations

$$\dot{u} = C_3(u, t)w + D_3(u, t) \tag{5}$$

where w is an $s \times 1$ array. In Eq. (4), π_p and γ_p, termed the *partial velocities*, are related to the kinematic coefficients by

$$\pi_p = \frac{\partial \boldsymbol{r}_p}{\partial u} C_3, \qquad \gamma_p = \frac{\partial \boldsymbol{r}_p}{\partial u} D_3 + \frac{\partial \boldsymbol{r}_p}{\partial t} \tag{6}$$

Choosing the generalized coordinates as $q = (\boldsymbol{R}, \Theta, u)$ and the velocity coordinates as $y = (\boldsymbol{V}, \boldsymbol{\omega}, w)$, the components of Eq. (1) are

$$C = \begin{bmatrix} C_1^T(\Theta) & 0 & 0 \\ 0 & C_2^{-1}(\Theta) & 0 \\ 0 & 0 & C_3(u, t) \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \\ 0 \\ D_3(u, t) \end{bmatrix} \tag{7}$$

Using any suitable method of dynamic formulation, such as Lagrange's [50], Gibbs [51], or Kane's equations [52–54], we arrive at the generalized mass array

$$M = \begin{pmatrix} m\dot{\boldsymbol{V}} - S\dot{\boldsymbol{\omega}} + \int_{\mathcal{D}} \rho\pi\dot{w}\,\mathrm{d}\mathcal{D} \\ S\dot{\boldsymbol{V}} + J\dot{\boldsymbol{\omega}} + \int_{\mathcal{D}} \rho\tilde{\boldsymbol{r}}\pi\dot{w}\,\mathrm{d}\mathcal{D} \\ \int_{\mathcal{D}} \rho\pi^T\,\mathrm{d}\mathcal{D}\dot{\boldsymbol{V}} - \int_{\mathcal{D}} \rho\pi^T\tilde{\boldsymbol{r}}\,\mathrm{d}\mathcal{D}\dot{\boldsymbol{\omega}} + \int_{\mathcal{D}} \rho\pi^T\pi\dot{w}\,\mathrm{d}\mathcal{D} \end{pmatrix} \tag{8}$$

where m is the total mass, ρ is the mass density, S and J are the first and second moments of inertia of the total system about F_B, and $\mathcal{D}$ is the geometric domain of the body. Note that, in general, S and J are not constant and are an explicit function of u (we note that determining these explicit functions will be a difficult problem for complex wing motions; curve fitting techniques may be an appropriate alternative). Also, if F_K is used instead of F_B, then by definition $S = 0$. Although this frame decouples the translational and rotational equations and simplifies the aerodynamic formulation, it is not necessarily a simplification of the dynamics because additional relative motions are introduced. The component of the generalized force that accounts for inertial forces becomes

$$\varphi^* = -\begin{pmatrix} m\tilde{\boldsymbol{\omega}} - \tilde{\boldsymbol{\omega}}S + \int_{\mathcal{D}} \rho\dot{\pi}w\,\mathrm{d}\mathcal{D} + 2\tilde{\boldsymbol{\omega}}\int_{\mathcal{D}} \rho\pi w\,\mathrm{d}\mathcal{D} \\ S\tilde{\boldsymbol{\omega}}\boldsymbol{V} - \tilde{\boldsymbol{\omega}}J\boldsymbol{\omega} + \int_{\mathcal{D}} \rho\boldsymbol{r}\dot{\pi}w\,\mathrm{d}\mathcal{D} + 2\int_{\mathcal{D}} \rho\tilde{\boldsymbol{r}}\,\tilde{\boldsymbol{\omega}}\,\pi w\,\mathrm{d}\mathcal{D} \\ \int_{\mathcal{D}} \rho\pi^T\,\mathrm{d}\mathcal{D}\tilde{\boldsymbol{\omega}}\boldsymbol{V} - \int_{\mathcal{D}} \rho\pi^T\tilde{\boldsymbol{\omega}}\,\tilde{\boldsymbol{r}}\,\mathrm{d}\mathcal{D}\boldsymbol{\omega} + \int_{\mathcal{D}} \rho\pi^T\dot{\pi}w\,\mathrm{d}\mathcal{D} + 2\int_{\mathcal{D}} \rho\pi^T\tilde{\boldsymbol{\omega}}\pi w\,\mathrm{d}\mathcal{D} \end{pmatrix} - \begin{pmatrix} \int_{\mathcal{D}} \rho\dot{\gamma}\,\mathrm{d}\mathcal{D} + 2\tilde{\boldsymbol{\omega}}\int_{\mathcal{D}} \rho\gamma\,\mathrm{d}\mathcal{D} \\ \int_{\mathcal{D}} \rho\tilde{\boldsymbol{r}}\,\dot{\gamma}\,\mathrm{d}\mathcal{D} + 2\int_{\mathcal{D}} \rho\tilde{\boldsymbol{r}}\,\tilde{\boldsymbol{\omega}}\,\gamma\,\mathrm{d}\mathcal{D} \\ \int_{\mathcal{D}} \rho\pi^T\dot{\gamma}\,\mathrm{d}\mathcal{D} + 2\int_{\mathcal{D}} \rho\pi^T\tilde{\boldsymbol{\omega}}\gamma\,\mathrm{d}\mathcal{D} \end{pmatrix} \tag{9}$$

For a collection of rigid bodies, we simply let $\pi = \mathbf{1}$ and $\gamma = 0$, so that

$$M = \begin{pmatrix} m\dot{\boldsymbol{V}} - S\dot{\boldsymbol{\omega}} + \ddot{\bar{S}} \\ S\dot{\boldsymbol{V}} + J\dot{\boldsymbol{\omega}} + \int_{\mathcal{D}} \rho\tilde{\boldsymbol{r}}\boldsymbol{v}\,\mathrm{d}\mathcal{D} \\ \bar{m}\,\dot{\boldsymbol{V}} - \bar{S}\,\dot{\boldsymbol{\omega}} + \ddot{\bar{S}} \end{pmatrix} \tag{10}$$

and

$$\varphi^* = -\begin{pmatrix} m\tilde{\boldsymbol{\omega}} - \tilde{\boldsymbol{\omega}}S + 2\tilde{\boldsymbol{\omega}}\,\dot{\bar{S}} \\ S\tilde{\boldsymbol{\omega}}\boldsymbol{V} - \tilde{\boldsymbol{\omega}}J\boldsymbol{\omega} + 2\int_{\mathcal{D}} \rho\tilde{\boldsymbol{r}}\,\tilde{\boldsymbol{\omega}}\,\boldsymbol{v}\,\mathrm{d}\mathcal{D} \\ \bar{m}\,\tilde{\boldsymbol{\omega}}\,\boldsymbol{V} - \tilde{\boldsymbol{\omega}}\,\bar{S}\,\boldsymbol{\omega} + 2\tilde{\boldsymbol{\omega}}\,\dot{\bar{S}} \end{pmatrix} \tag{11}$$

where $\bar{m}$ is the mass and $\bar{S}$ the first inertial moment, defined about F_B, of bodies moving relative to the main body. If desired, the relative motions of each rigid may be expanded into local coordinate frame representations, most conveniently located at the center of mass of each body [39]; this entails π being composed of orthogonal transformation operators between the basis vectors of relative frames (see, for example, [54,55]).

The applied force takes the form

$$\varphi^* = \begin{pmatrix} \int_{\mathcal{D}} \boldsymbol{F}\,\mathrm{d}\mathcal{D} + mgC_1[0 \quad 0 \quad 1]^T \\ \int_{\mathcal{D}} \tilde{\boldsymbol{r}}\boldsymbol{F}\,\mathrm{d}\mathcal{D} + mgSC_1[0 \quad 0 \quad 1]^T \\ \int_{\mathcal{D}} \pi^T\boldsymbol{F}\,\mathrm{d}\mathcal{D} \end{pmatrix} \tag{12}$$

where g is the gravitational acceleration, and $\boldsymbol{F}$ is the force distribution due to aerodynamic loads, control surfaces deflections, and engine thrust. Internally generated forces are those due to friction at the connection joints of the bodies ($\boldsymbol{\tau}_f$) and controlled internal actuation ($\boldsymbol{\tau}_a$):

$$\tau = \begin{pmatrix} \int_{\mathcal{D}} (\boldsymbol{\tau}_f + \boldsymbol{\tau}_a)\,\mathrm{d}\mathcal{D} \\ \int_{\mathcal{D}} \tilde{\boldsymbol{r}}(\boldsymbol{\tau}_f + \boldsymbol{\tau}_a)\,\mathrm{d}\mathcal{D} \\ \int_{\mathcal{D}} \pi^T(\boldsymbol{\tau}_f + \boldsymbol{\tau}_a)\,\mathrm{d}\mathcal{D} \end{pmatrix} \tag{13}$$

All of the terms in Eqs. (8) and (9) exist for the conventional "rigid" aircraft due to the motion of the control surfaces and elastic deformations. In most cases the control surfaces are of negligible mass and contribute very little to the dynamics. In the case of large structural morphing, the contribution of these additional effects (relative to the applied forces) will be dependent on the mass of the structure, the rate of change, and the main body motions. We also note that it is possible that several terms of the dynamics equations may be disregarded, depending on the flight conditions. For instance, in the derivation of the short-period longitudinal equations, the terms mU_0 (where U_0 is the forward aircraft velocity) and $2S_x$ (where S_x is the x component of S) appear in the angle-of-attack equations. For typical aircraft operation $2S_x \ll mU_0$, and thus $2S_x$ can be disregarded.

B. Specified Motions

For design purposes we often do not wish to model every detail of the aircraft design, particularly friction caused by mechanical constraints. It is more convenient to assume that a structural control system provides adequate performance given some desired configuration state. The simplest method to account for internal forces is by partially specified motion. For a constrained dynamic system, τ is of the form

$$\tau = B^T \lambda \tag{14}$$

where λ is a $m \times 1$ array of Lagrange multipliers. It is assumed that the constraints can be expressed in terms of the kinematic relations as

$$B\dot{q} = v \tag{15}$$

where v is a $m \times 1$ array possibly dependent on q and t. As such, the equations

$$B\ddot{q} = \dot{v} - \dot{B}\,\dot{q} \tag{16}$$

must be appended to the dynamic equations. Then by multiplying Eq. (2) by an orthogonal complement of B, denoted $B^\perp$ where by definition $BB^\perp = 0$, the constraints are removed from dynamic equations [54,56]. In reality this assumption may be realized by defining a control system, shown in Fig. 3, resulting in closed-loop dynamics in the form of a second-order differential equation. By this process we are able to regard mechanism control as a separate problem. To accommodate specified motion, suppose that $\dot{u} = v$; then a sufficient choice for the orthogonal complement is

$$B^\perp = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T \tag{17}$$

By multiplying M and f by $(B^\perp)^T$ and appending the constraint equations, τ goes to zero and we replace the third row of Eqs. (8), (9), and (12), by

$$C_3\dot{w} = -\dot{C}_3 w - \dot{D}_3 + \dot{v} \tag{18}$$

Additionally, the controlled structural equations must be defined. Assuming the controller compels second-order behavior, we have

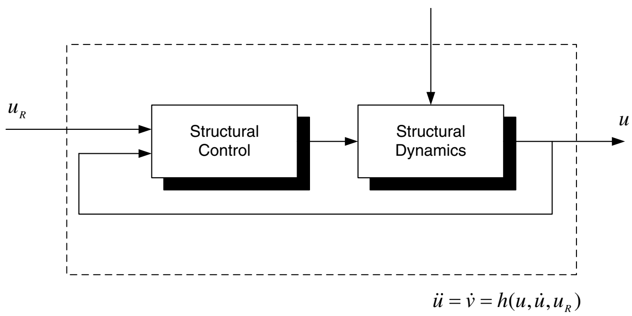

Fig. 3 Constraining the dynamic equations by structural feedback control.

$$\dot{v} = h(u, \dot{u}, u_R) \tag{19}$$

where u_R is the reference input.

These types of constraints, where the motions are specified, are generally termed *program constraints* or *servoconstraints*. An interesting problem that stems from partly specified motion is that of determining the actuation forces required to achieve the desired motions [56–58]. Here we do not consider such problems; that is, the program constraints are achieved independent of external influence.

IV. Aerodynamic Modeling

For a body moving through the Earth's atmosphere, the generalized applied force φ includes contributions from gravity, aerodynamic forces, and any thrust supplied by the body:

$$\varphi = \varphi_g(q) + \varphi_a(q, y, \eta, t) + \varphi_T \tag{20}$$

where η represents the atmospheric conditions (altitude, air speed, etc.). Constructing an analytical function φ_a is, at best, a very difficult problem due to the complex functional dependency. Approximate representations for such functions typically derive from a combination of aerodynamic theory, experimentation, and computational analysis. To numerically simulate atmospheric flight dynamics beyond a small perturbation linear regime, φ_a is often approximated by a discrete lookup table that maps an input (q, y, and η) to a steady-state force and moment couple applied to the aircraft's center of mass; this is why F_K is often an important frame of reference. Aerodynamic computer codes, such as Datcom, can be used to populate such a lookup table. Although, these numerical algorithms analyze the wings and fuselage separately, complex blending methods are often required to accurately describe the total aerodynamic contribution. It is for this reason that employing multiple reference frames in formulating the dynamic equations can be troublesome, because this would require a determination of the aerodynamic force on each member and then make corrections for interaction effects.

For purposes of model-based control, it is highly desirable that the aerodynamic forces be formulated as time-invariant state-dependent functions. For rigid aircraft such functions are realized through standard buildup methods that incorporate the aerodynamic derivatives [59–61]. Given a nominal state (q_0, y_0, η_0), where η_0 is assumed to remain fixed, the generalized aerodynamic force is of the form

$$\varphi_a(q, y, \eta, t) \approx N(q, y, \eta_0) \cong N_0(q_0, y_0, \eta_0) + N_q(q_0, \eta_0)\Delta q + N_y(y_0, \eta_0)\Delta y \tag{21}$$

where $N_q = \partial N/\partial q$ and $N_y = \partial N/\partial y$. For morphing applications, where large planform changes are implemented, we would like to have a similar expression for the generalized force. However, the extension of this technique to morphing is not readily apparent. Certainly it is possible to construct a function N_u for every planform shape (quantified by the coordinate array u, contained in q) and at every flight configuration. Besides being extremely time consuming this approach also neglects transient effects which may be significant.

First-order panel methods lend themselves to developing state-dependent aerodynamic forces. A relatively simple method for deriving analytical functions is blended modeling that incorporates empirical functions from Datcom and circulation distributions from computational algorithms like vortex-lattice or vortex-ring methods [62]. Iteration through a planform trajectory results in smooth aerodynamic functions that can be exploited for first-cut stability analysis and control design. In general, it is possible to develop a set of quasisteady aerodynamic functions that can be expanded about the instantaneous planform coordinates:

$$\varphi_a(q, y, \eta, t) \approx N[q(t), y(t), \eta_0] \cong N_0[q_0(t), \eta_0] + N_q[q_0(t), \eta_0]\Delta q(t) + N_y[y_0(t), \eta_0]\Delta y(t) \tag{22}$$

This is the equivalent of developing a stability derivative for the planform variation and updating the expansion point at each instant in time. For small perturbations in the aerodynamic state we can represent the general aerodynamic function as

$$N[q(t), y(t)] \approx \bar{N}[q(t)]y(t) \tag{23}$$

such that the planform state derivative in Eq. (22) becomes

$$N_q = \frac{\partial \bar{N}}{\partial q}[q_0(t), \eta_0]y(t) \tag{24}$$

resulting in

$$\varphi_a(q, y, t) \approx N_0[q_0(t), \eta_0] + \frac{\partial}{\partial q}\bar{N}[q_0(t), \eta_0]y(t)\Delta q(t) + \bar{N}[q_0(t), \eta_0]y(t) \tag{25}$$

As an example, applying Eq. (25) to the pitching moment yields

$$C_M(u, \alpha, Q, t) \approx C_{M_0}[u_0(t)] + \left[1 + \Delta u(t)\frac{\partial}{\partial u}\right][C_{M_\alpha}u_0(t)\alpha(t) + C_{M_q}u_0(t)Q(t)] \tag{26}$$

Equation (26) requires the standard aerodynamic coefficients and stability derivatives as class $\mathcal{C}^1$ differentiable functions of the planform state. Overall, we are interested in the stability and performance of transient planform motion within a finite time span at a transition rate much less than the freestream velocity. Equation (26) is a quasisteady representation but does not include explicit unsteady coefficients. Developing aerodynamic forces as a function of time and using a Taylor expansion to obtain an augmented set of stability derivatives is an admittedly crude method of incorporating motion time history into the flight equations. This is commonly used for the angle-of-attack derivative in the standard stability equations. The method is sufficiently accurate when the change in aircraft state is "slow enough" for the flowfield to realign at each time increment. For classic unsteady motions such as a variable wing incidence, analytical bounds on input velocity have been established for the application of "unsteady" stability derivatives such as angle-of-attack rate derivatives. Such bounds have not been developed for a variable planform, but by constraining the planform rate we can use time-augmented stability derivatives as a first approximation. The challenge then becomes development of the time-varying forces for a variable planform state.

By nature, the panel methods in the hybrid modeling assume quasisteady planform states but can be upgraded to include unsteady effects. This can be accomplished with minimal complexity by substituting the steady-state flow boundary conditions with instantaneous flow constraints (taken from a time-varying boundary function) corresponding to the relative motion of the flowfield [63]. The result is a time-varying circulation distribution and transient aerodynamic forces required for a Taylor expansion about the planform coordinate u and its derivative $\dot{u}$,

$$C_M(u, \alpha, Q, t) \approx \left[1 + \Delta u(t)\frac{\partial}{\partial u} + \Delta\dot{u}(t)\frac{\partial}{\partial\dot{u}}\right][C_{M_\alpha}u_0(t)\alpha(t) + C_{M_q}u_0(t)Q(t)] + C_{M_0}u_0(t) \tag{27}$$

in which there is implicit time dependence through u and $\dot{u}$. A noted difficulty is that iteration of an unsteady panel code is required through, not only static planform states, but also for various $\dot{u}$s.

The augmented stability derivative representation is suitable for initial control implementation but the wake history should be tracked over the planform transition time for accurate analysis and increased control effectiveness. There is no developed wake model for variable planform inputs and is the subject of current research. Attempting to compute the wake history numerically and model the response with lookup tables or neural networks is not a consistently predictable method. A modern technique such as state-space modeling has shown promise when compared with experimental results and is a preferred method [64]. State-space modeling incorporates a nonlinear mapping between a static flow characteristic and internal state variables that describe the time-varying flowfield. Research on the application of these methods to dynamic motions orthogonal to the flow direction is required to develop appropriate models.

An alternative analysis method is to relax the aerodynamic modeling in favor of developing a robust control mechanism. The deviation in unsteady response can be bounded by some gain k of the quasisteady aerodynamic coefficients such that

$$\varphi_a(q, y, t) \leq k|N(q)|y \tag{28}$$

Wind tunnel analysis could be used to parameterize k in the morphing variables to aid in control design.

V. Principles of Flight Control

Because aircraft control is model based, and the aerodynamic forces can change significantly and with a degree of uncertainty, an enormous amount of information is required in the design process. For a rigid aircraft the implementation of a flight control system involves an extensive process of aerodynamic testing, analytical design, numerical simulation, and flight testing. For a multi-configuration aircraft it is apparent that the task of control design becomes more involved, in that a robust, optimized design is required for each operating point and for each aircraft configuration. The design process then becomes equivalent to developing a flight control system for several different aircraft types.

In addition to rigid-body flight there is also the consideration of the transient phase of morphing, that is, control of the aircraft during the transition from one structural state to the next. Although modern aircraft certainly contain various moving components, in the majority of cases the effect of these components can be considered negligible due to small displacements and low relative mass. As a result, the inertial forces caused by the moving components can be approximated as constant without significant consequence. For an aircraft undergoing large structural changes, however, standard rigid-body analysis is insufficient. If the changes are slow, then a quasisteady dynamic and aerodynamic approximation may be sufficient. If, alternatively the changes are desired to be very fast or while maneuvering, and the moving components are of substantial mass, then it is uncertain whether this assumption is sufficient for ensuring stable and high-performance flight.

A. State Representation

In consideration of Eq. (2), it is not possible to place the general form of the dynamic equations into that of a standard state representation $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$. This is a severe disability because most control theory is developed from a state-space representation of systems. In this respect, it is not convenient to consider the morphing structures as continuous elements, but rather composed of a finite collection of masses; for instance, the integral form $\int_{\mathcal{D}} \rho\pi\dot{w}\,\mathrm{d}\mathcal{D}$ is replaced by the discrete form $(\sum_{p=1}^{\nu} m_p\pi_p)\dot{w}$ and so forth. Also, from Eq. (4), it is necessary to let $\pi_p = 1$ and $\gamma_p = 0$ for all $p = 1, \ldots, \nu$. Given these stipulations and certain assumptions regarding the aerodynamic behavior, the equations of motion for a morphing aircraft can be expressed in the general form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) + \delta\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \tag{29}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ is the control input, and $\delta\boldsymbol{f}$ is a function variation that allows for model uncertainty. For brevity we will ignore the output state vector that indicates those states available for feedback. The state equations will also depend on the local aerodynamic conditions, but we do not explicitly define the dependence. Using previous notation, the state vector can be defined as

$$\boldsymbol{x} = [\boldsymbol{R}^T \quad \Theta^T \quad u^T \quad \boldsymbol{V}^T \quad \boldsymbol{\omega}^T \quad w^T]^T \tag{30}$$

and the control input as

$$u = [\delta^T \quad T^T \quad u_R^T]^T \tag{31}$$

where $\boldsymbol{\delta}$ denotes the aerodynamic control effectors, and $\boldsymbol{T}$ the thrust. We may also choose other states such as error states for tracking control, integral states for eliminating steady-state error, and/or those states corresponding to convenient frames of reference such as the stability or wind axes [59]. Also, the input vector may contain tracking commands. Note that if it were not for the morphing states u and w, it would be possible to formulate the state representation in the more approachable form

$$\dot{x} = f(x) + \delta f(x) + [G(x) + \delta G(x)]u \tag{32}$$

Since the closed-loop structural dynamics are of the form $\ddot{u} = h(u, \dot{u}, u_R)$, it is not possible to construct Eq. (32) unless h exhibits linear dependence in u_R. For complex wing motions this may not be possible.

B. Control Approaches

There are several approaches to controlling a large-scale morphing aircraft, some of which are encompassed in the diagram of Fig. 4. The first approach is to change shape in an open-loop manner based on some a priori estimate of long-term vehicle performance such as range, speed, or maneuverability. The structural control operates independent of the instantaneous aircraft state, without regards to stability or short-term performance. This amounts to defining u_R in Eq. (31) as an explicit function of time. The objective of the flight control design is then stability and performance in spite of the structural changes.

A second approach is to define the additional error state $e_u = u - u_d$, where u_d is the desired planform shape. Here u_d is regarded as an additional command input (preferably a constant input), again generated by some online performance metric. This formulation provides a means of assessing flight stability and performance since u_R will be designed alongside the remaining control inputs. As a result, the planform alterations will aid in the stabilization of the aircraft taking into account both the inertial and aerodynamic forces produced. Here we are again confronted with a troublesome form of the error dynamics given by

$$\ddot{e}_u = h(e_u, \dot{e}_u, u_R, u_d) \tag{33}$$

where, in general, it is not possible to extract u_R and u_d from h. As discussed by Khalil [65], the most practical approach to such problems is to approximate the dynamics through linearization or to linearize the dynamics through feedback control.

A third approach is to consider the structural changes as additional inertial/aerodynamic forcing inputs that are available both in short-term maneuvering and long-term performance. In this scheme, structural changes are determined by the control algorithm and there is no explicitly defined input except for a spacial tracking command, provided by the navigation system. This type of control is similar to that found in nature. We note that, besides morphing applications, researchers have begun to investigate making use of inertial forces to obtain better control authority. Moving mass control has been explored for underwater vehicles [66,67], space structures [68,69], kinetic warheads [70], and reentry vehicles [71]. The main difference between these investigations and the current is that external forces can be essentially decoupled from those internally generated.

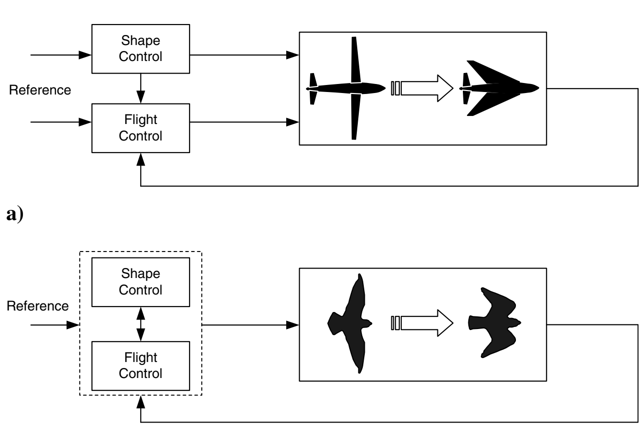

Fig. 4 Methods of morphing flight control: a) independent shape and flight control, b) integrated shape and flight control (birdlike flight).

C. Linear Approximations

Because it is often difficult to work directly with the full nonlinear equations it is convenient, and sometimes necessary, to consider the reduced set of equations furnished by the longitudinal, lateral, or steady-state flight assumptions (i.e., steady-wings level, steady turning, steady pull-up, steady roll, etc). Upon reducing the state and control vector ($\boldsymbol{x} \to \boldsymbol{z}$ and $\boldsymbol{u} \to \boldsymbol{w}$) and linearizing about some specific operating point $(z_0, \boldsymbol{w}_0, t_0)$, the state equations are of the form

$$\dot{z} = (A + \delta A)z + (B + \delta B)w \tag{34}$$

We assume that the aerodynamic forces are incorporated into the flight equations via the quasisteady assumption which results in the aerodynamic derivatives that populate the constant matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. As such, any transient aerodynamic effects must be accounted for in the variations of $\delta \boldsymbol{A}$ and $\delta \boldsymbol{B}$.

By an appropriate selection of operating conditions the morphing dynamics can be characterized by the linear-parameter-varying (LPV) system

$$\dot{z} = A(z)z + G(z)w \tag{35}$$

where the perturbation terms have been ignored for convenience. The form of Eq. (35) is also that used for high-performance rigid aircraft with a range of operation that occupies a large region of the state space. For the rigid case, the linearization points chosen for analysis are taken from the set of all possible z to cover the regions of the state space that correspond with the varying operational environments. The introduction of morphing increases the dimension z by $m + s$ (m being the dimension of u and s the dimension of w). Additionally, the choice of operating points requires specifying the region of the state space where morphing will be implemented, specifically, whether morphing will be allowed during maneuvering or limited to, for instance, steady wings-level flight. Although it might be argued that it is not necessary to maneuver while changing shape, one could certainly envision a situation where it is imperative to arrive at a destination in the least possible time.

The other state model of interest is one that allows for aerodynamic forces that are given as an explicit function of time, with the assumption that $u(t_i) := u_i$ is approximately known to within Δu_i at any t_i in some interval. This amounts to performing a time-dependent planform change in which, if tracking is sufficient, we can estimate the aerodynamic forces for each point of interest. If the equations are linearized at appropriate values of $(z_0, \boldsymbol{w}_0, t_0)$, then we can consider the morphing dynamics by the linear-time-varying (LTV) system

$$\dot{z} = A(t)z + G(t)w \tag{36}$$

Here the process of selecting the linearization points is different than for the LPV system, because the space from which they are chosen is narrowed to the intersection of the sets of all z and u_i (i.e, $\{z\} \cap \{u_i\}$).

Linearization naturally assumes that the resulting state dynamics $z(t)$ are a reasonable estimation to within some acceptable variation $\delta z(t)$. During the process of morphing, the magnitude (norm) of this variation is likely reduced significantly compared to a fixed-wing aircraft. For example, while $\boldsymbol{A}$ in Eq. (34) may be approximately constant for small changes in the angle of attack about a steady-wings-level stationary point, even small increments in u can have significant effects on the state dynamics. Therefore, in control design one may either select more operating points to enhance the accuracy of the nominal model, or accommodate the uncertainty through $\delta \boldsymbol{A}$

and δG; the latter approach will ultimately lead to a more conservative control design. In stabilization applications such as pitch-attitude hold, altitude/Mach hold, roll angle hold, etc., a conservative design is justified. Alternatively, in maneuvering flight both stability and performance are required.

D. Control Methods

There are numerous methods for designing robust, high-performance controllers given the linear time-invariant (LTI) dynamics of Eq. (34); several of these can be found in the texts of Zhou [72] and, with specific regards to aircraft applications, Stevens and Lewis [61]. The extension of these methods to morphing transition flight is conceptually straightforward. The basic concept is to employ linear methods that are optimized at various operating points. The process of updating the control gains to correspond with the operating point of interest can be accomplished by *gain scheduling*.

Perhaps the most common conventional method of controlling modern aircraft, and nonlinear systems in general, gain scheduling integrates multiple linear control designs to provide adequate performance over a range of operating conditions. Gain scheduling applied to LPV and LTV systems is fundamentally different, most notably because the scheduling parameter chosen for the LPV system (not necessarily a member of $\boldsymbol{x}$) must be measured by a sensor; here the additional complexity is apparent. In theory, if a LPV system can be sufficiently approximated as a collection of parameter invariant linear systems, a set of controllers that stabilize the individual linear systems can be scheduled to stabilize the entire operating region provided that the scheduled variable is sufficiently slow varying [73,74]. However, performance and global stability of the gain-scheduled design over the entire operating range is more often assumed rather than implied [75]. Alternatively, scheduling parameters for the LTV system are largely established by the selected time increments $\{t_i\}$, and thus require no sensors.

The concept of gain scheduling is simple; however, application is typically not. In the application of gain scheduling there are three considerations that must be addressed: 1) selection of scheduling points, 2) selection of the scheduling parameter, and 3) the scheduling algorithm. As noted by Rugh [73], the selection of scheduling points is typically based on the particular application and with the insight gained from experience. For LPV systems, the dictum is "schedule on a slow variable"; to date there is not an established procedure for quantifying this statement. The interpolating scheme has also been a source of much investigation; a brief overview was given by Stilwell [76].

For rigid aircraft applications there are some established guidelines for gain scheduling. For instance, the scheduling for a pitch-attitude hold controller can be performed by attempting to maintaining a constant ratio $-K_\theta C_{M_{\delta_e}}/C_{M_\alpha}$ by updating the proportional gain K_θ [77]. This, however, relies on the assumption that the aerodynamic derivatives do not alter substantially. Therefore, it is not apparent whether such existing guidelines carry over to morphing applications. With regards to the scheduling parameter, an intuitive approach might be to use motion of the center of mass at which the nominal control design will be designed. It is uncertain whether this is appropriate and is likely to depend on the operating conditions. Secondly, the center of mass position must be accurately sensed, which can be a difficult task.

Another method of control that is potentially applicable to the state dynamics of Eq. (29) or Eq. (34) is the so-called Lyapunov based-control design (or simply nonlinear control). Nonlinear control design relies on Lyapunov's second method in which a Lyapunov function is used to derive a stabilizing control law. In general, finding the required Lyapunov function is a nontrivial task. One way of avoiding this problem is to employ a method of differential geometry, termed *feedback linearization*, in which the dynamic equations are placed in a form where finding the required Lyapunov function is trivial. In the flight control community this is termed *dynamic inversion* and has been of particular interest over the past several years [78–82].

In applying the dynamic inversion control design, the most important considerations are the *matching condition* and stability of the *zero dynamics* [65,83,84]. The method first requires that state representation be of the form, or capable of transformation to the form, of Eq. (32). The matching condition then requires that uncertain terms be attached solely to the control state vector; that is, $\delta \boldsymbol{f}(\boldsymbol{x}) = 0$. It is important because there are several methods of guaranteed stability given that the condition is met. As demonstrated by the state representations of Eqs. (29) and (34), although they may be approximated as such, the flight dynamics never satisfy the matching condition. Matching conditions have been relaxed for certain limited classes of systems. For example, Qu and Dawson demonstrated robust linearization given a system that does not meet the matching condition, but can be represented as a cascaded system where each subsystem individually satisfies the condition [85]. The flight dynamic equations of morphing do not fall under any of these categories. The zero dynamics are the state dynamics left unobservable by the control design. In many cases the assumption of stable zero dynamics is necessary due to nonlinearity of the resulting equations. Linearization and numerical simulations are both methods of investigating the stability of the zero dynamics.

For an ideal case, when the physical model is in controller canonical form, robust techniques such as a sliding mode control can be easily implemented. Methods such as the *Lyapunov redesign* attempt to add additional control input to the system based on the bounds of the matched error. The forward approach to nonlinear design, without feedback linearization, is to design a controller and then determine a function that satisfies the Lyapunov criterion for the closed-loop system. Such an approach, however, often results in a conservative design because there is no direct means of assessing performance. For linear systems, performance can be neatly evaluated by a quadratic performance index [61].

E. Discussion

For the case of manned aircraft or remotely piloted vehicles, the pilot provides the additional sensory and learning capabilities for improving performance. However, the pilot does not have a sufficient means of updating the control system directly and must rely on the communication of mostly qualitative information to the flight control designer. The flight control designer must compensate for a lack of sensory feedback by implementing algorithms based upon predictive models. These models are, of course, only an approximation of true behavior. This fact is both inherent and necessary; it is necessary because a predictive model must be simple enough to facilitate design.

In contrast, birds and flying insects have multiple and highly sensitive mechanisms for sensing speed, direction, orientation, and wing/air interactions. Comparatively, the flight control sensors for aircraft are relatively few and coarse. Birds and insects are able to rely heavily upon the vast amount of sensory input for feedback, thus allowing a process of constant learning and updating of the control system. Although the exact process of how this occurs is not well understood, it allows for both optimized long-term flight and radical short-term maneuvering. The latter includes both symmetric and asymmetric wing movement and deformation to take advantage of both inertial and aerodynamic effects.

As material and sensing technology advances we will begin to approach the flight capabilities produced by nature. What remains is a control method to accommodate multiple sensors and actuators, and more generally, the complexity that will inevitably appear in these systems. In this regard, another method of control that was not discussed is the largely undeveloped method of adaptive control. Historically, adaptive techniques have not been in good standing with the flight control community (this is not without good reason; see, for example [86]). This view possibly began with the crash of the X-15, which was the first aircraft tested with an adaptive autopilot. The skepticism associated with adaptive control will, without doubt, be overshadowed by the implications involved [87]. The implications of adaptive control are robust and high-performance flight without complete specification of the model nor of the model uncertainties. Both can be learned and adapted to. Although at the current time, an overwhelming majority of operational aircraft continue to employ conventional linear methods of control design, more advanced techniques will ultimately be required for ideal performance of morphing aircraft flight.

VI. Conclusions

The field of morphing research is composed of a large array of interdisciplinary studies. The part of this research dedicated to flight control contains the typically disparate fields of nonrigid body dynamics, aerodynamic theory, and nonlinear control theory. Although it was demonstrated that several methods exist for accommodating all aspects of this field, due to the underlying complexity involved none are especially ideal. In particular, although there are highly developed rules and procedures to accommodate standard rigid aircraft designs, the extension of these methods to morphing flight is not at all apparent. In general, one may either conform the dynamic equations to accommodate existing methods of rigid aircraft control, or design a controller to accommodate the dynamics. The former approach provides more developed tools, such as the extensive literature of linear system theory. The latter approach, although more involved, would provide a means to take advantage of shape change for both long-term benefits such as increased range and short-term maneuvering and stabilization, thus bringing morphing flight closer to optimal performance.

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